3.5 1) $f(x) = \dfrac{12-2\,x}{3-x}$ n'est pas définie si $3-x=0$, c'est-à-dire si $x=3$.

(a) $\displaystyle\lim_{\substack{x\to 3\\ x<3}} \frac{12-2\,x}{3-x} = \frac{6}{0_+} = +\infty$

(b) $\displaystyle\lim_{\substack{x\to 3\\ x>3}} \frac{12-2\,x}{3-x} = \frac{6}{0_-} = -\infty$

2) $f(x) = \dfrac{x^2+3\,x+2}{x^2+4\,x+4}$ n'est pas définie si $x^2+4\,x+4 = (x+2)^2 = 0$, à savoir si $x=-2$.

(a) $\displaystyle\lim_{\substack{x\to -2\\ x<-2}} \frac{x^2+3\,x+2}{x^2+4\,x+4} = \frac{0_-}{0_+}$: indéterminé

$\displaystyle\lim_{\substack{x\to -2\\ x<-2}} \frac{x^2+3\,x+2}{x^2+4\,x+4} = \lim_{\substack{x\to -2\\ x<-2}} \frac{(x+1)\,(x+2)}{(x+2)^2} = \lim_{\substack{x\to -2\\ x<-2}} \frac{x+1}{x+2} = \frac{-1}{0_-} = +\infty$

(b) $\displaystyle\lim_{\substack{x\to -2\\ x>-2}} \frac{x^2+3\,x+2}{x^2+4\,x+4} = \lim_{\substack{x\to -2\\ x>-2}} \frac{(x+1)\,(x+2)}{(x+2)^2} = \lim_{\substack{x\to -2\\ x>-2}} \frac{x+1}{x+2} = \frac{-1}{0_+} = -\infty$

3) $f(x) = \dfrac{x^2+x-2}{(x-1)^2}$ n'est pas définie si $(x-1)^2=0$, c'est-à-dire si $x=1$.

(a) $\displaystyle\lim_{\substack{x\to 1\\ x<1}} \frac{x^2+x-2}{(x-1)^2} = \frac{0_-}{0_+}$: indéterminé

$\displaystyle\lim_{\substack{x\to 1\\ x<1}} \frac{x^2+x-2}{(x-1)^2} = \lim_{\substack{x\to 1\\ x<1}} \frac{(x+2)\,(x-1)}{(x-1)^2} = \lim_{\substack{x\to 1\\ x<1}} \frac{x+2}{x-1} = \frac{3}{0_-} = -\infty$

(b) $\displaystyle\lim_{\substack{x\to 1\\ x>1}} \frac{x^2+x-2}{(x-1)^2} = \lim_{\substack{x\to 1\\ x>1}} \frac{(x+2)\,(x-1)}{(x-1)^2} = \lim_{\substack{x\to 1\\ x>1}} \frac{x+2}{x-1} = \frac{3}{0_+} = +\infty$

4) $f(x) = \dfrac{x^2+6\,x-7}{-x^3+x^2-x+1}$ n'est pas définie si $-x^3+x^2-x+1 =$
$x^2+1-x^3-x = (x^2+1) - x\,(x^2+1) = \underbrace{(x^2+1)}_{\geqslant 1}(1-x) = 0$

à savoir si $x=1$.

(a) $\displaystyle\lim_{\substack{x\to 1\\ x<1}} \frac{x^2+6\,x-7}{-x^3+x^2-x+1} = \frac{0_-}{0_+}$: indéterminé

$\displaystyle\lim_{\substack{x\to 1\\ x<1}} \frac{x^2+6\,x-7}{-x^3+x^2-x+1} = \lim_{\substack{x\to 1\\ x<1}} \frac{(x-1)\,(x+7)}{(x^2+1)\,\underbrace{(1-x)}_{-(x-1)}} = \lim_{\substack{x\to 1\\ x<1}} -\frac{x+7}{x^2+1} =$
$\displaystyle -\frac{8}{2} = -4$

(b) De même, $\displaystyle\lim_{\substack{x\to 1\\ x>1}} \frac{x^2+6\,x-7}{-x^3+x^2-x+1} = -4$

5) $f(x) = \dfrac{\sqrt{x+24} - \sqrt{x+15}}{x-1}$ est définie si

- $x + 24 \geqslant 0 \iff x \geqslant -24$
- $x + 15 \geqslant 0 \iff x \geqslant -15$
- $x - 1 \neq 0 \iff x \neq 1$

C'est pourquoi $\mathrm{D}_f = [-15\,;1[\cup]1\,;+\infty[$.

(a) $\displaystyle\lim_{\substack{x\to1\\x<1}} \frac{\sqrt{x+24} - \sqrt{x+15}}{x-1} = \frac{1}{0_-} = -\infty$

(b) $\displaystyle\lim_{\substack{x\to1\\x>1}} \frac{\sqrt{x+24} - \sqrt{x+15}}{x-1} = \frac{1}{0_+} = +\infty$

6) $f(x) = \dfrac{x + \sqrt{x+6}}{x^2 + 5\,x + 6}$ est définie si

- $x + 6 \geqslant 0 \iff x \geqslant -6$
- $x^2 + 5\,x + 6 = (x+2)\,(x+3) \neq 0 \iff x \neq -2$ et $x \neq -3$

Il en résulte que $\mathrm{D}_f = [-6\,;-3[\cup]-3\,;-2[\cup]-2\,;+\infty[$.

(a) $\displaystyle\lim_{\substack{x\to-3\\x<-3}} \frac{x + \sqrt{x+6}}{x^2 + 5\,x + 6} = \frac{-3+\sqrt{3}}{0_+} = -\infty$

(b) $\displaystyle\lim_{\substack{x\to-3\\x>-3}} \frac{x + \sqrt{x+6}}{x^2 + 5\,x + 6} = \frac{-3+\sqrt{3}}{0_-} = +\infty$

(c) $\displaystyle\lim_{\substack{x\to-2\\x<-2}} \frac{x + \sqrt{x+6}}{x^2 + 5\,x + 6} = \frac{0_-}{0_-}$: indéterminé

$$\lim_{\substack{x\to-2\\x<-2}} \frac{x + \sqrt{x+6}}{x^2 + 5\,x + 6} = \lim_{\substack{x\to-2\\x<-2}} \frac{(x + \sqrt{x+6})\,(x - \sqrt{x+6})}{(x+2)\,(x+3)\,(x - \sqrt{x+6})} =$$
$$\lim_{\substack{x\to-2\\x<-2}} \frac{x^2 - (x+6)}{(x+2)\,(x+3)\,(x - \sqrt{x+6})} = \lim_{\substack{x\to-2\\x<-2}} \frac{(x+2)\,(x-3)}{(x+2)\,(x+3)\,(x - \sqrt{x+6})} =$$
$$\lim_{\substack{x\to-2\\x<-2}} \frac{x-3}{(x+3)\,(x - \sqrt{x+6})} = \frac{-5}{-4} = \frac{5}{4}$$

(d) De même, $\displaystyle\lim_{\substack{x\to-2\\x>-2}} \frac{x + \sqrt{x+6}}{x^2 + 5\,x + 6} = \frac{5}{4}$