3.6 On rappelle que $|x| = \begin{cases} x & \text{si } x \geqslant 0 \\ -x & \text{si } x < 0 \end{cases}$

1) (a) $\displaystyle\lim_{\substack{x\to 0\\ x<0}} \frac{x^2+|x|}{|x|} = \lim_{\substack{x\to 0\\ x<0}} \frac{x^2-x}{-x} = \lim_{\substack{x\to 0\\ x<0}} \frac{x\,(x-1)}{-x} = \lim_{\substack{x\to 0\\ x<0}} 1-x = 1$

(b) $\displaystyle\lim_{\substack{x\to 0\\ x>0}} \frac{x^2+|x|}{|x|} = \lim_{\substack{x\to 0\\ x>0}} \frac{x^2+x}{x} = \lim_{\substack{x\to 0\\ x>0}} \frac{x\,(x+1)}{x} = \lim_{\substack{x\to 0\\ x>0}} x+1 = 1$

2) (a) $\displaystyle\lim_{\substack{x\to 2\\ x<2}} \frac{x+1}{x^2-4} = \lim_{\substack{x\to 2\\ x<2}} \frac{x+1}{(x-2)\,(x+2)} = \frac{3}{0_-} = -\infty$

(b) $\displaystyle\lim_{\substack{x\to 2\\ x>2}} \frac{x+1}{x^2-4} = \lim_{\substack{x\to 2\\ x>2}} \frac{x+1}{(x-2)\,(x+2)} = \frac{3}{0_+} = +\infty$

3) (a) $\displaystyle\lim_{\substack{x\to 1\\ x<1}} \frac{x}{x^2-2\,x+1} = \lim_{\substack{x\to 1\\ x<1}} \frac{x}{(x-1)^2} = \frac{1}{0_+} = +\infty$

(b) $\displaystyle\lim_{\substack{x\to 1\\ x>1}} \frac{x}{x^2-2\,x+1} = \lim_{\substack{x\to 1\\ x>1}} \frac{x}{(x-1)^2} = \frac{1}{0_+} = +\infty$

4) (a) $\displaystyle\lim_{\substack{x\to 0\\ x<0}} \frac{x^2-2\,x}{|x|} = \lim_{\substack{x\to 0\\ x<0}} \frac{x^2-2\,x}{-x} = \lim_{\substack{x\to 0\\ x<0}} \frac{x\,(x-2)}{-x} = \lim_{\substack{x\to 0\\ x<0}} 2-x = 2$

(b) $\displaystyle\lim_{\substack{x\to 0\\ x>0}} \frac{x^2-2\,x}{|x|} = \lim_{\substack{x\to 0\\ x>0}} \frac{x^2-2\,x}{x} = \lim_{\substack{x\to 0\\ x>0}} \frac{x\,(x-2)}{x} = \lim_{\substack{x\to 0\\ x>0}} x-2 = -2$