3.7 1) $\lim\limits_{x\to 1} \dfrac{x^2-2\,x+1}{x-1} = \dfrac{0}{0}$: indéterminé

$\lim\limits_{x\to 1} \dfrac{x^2-2\,x+1}{x-1} = \lim\limits_{x\to 1} \dfrac{(x-1)^2}{x-1} = \lim\limits_{x\to 1} x-1 = 0$

2) $\lim\limits_{x\to 5} \dfrac{x^3-3\,x+2}{x^2-6\,x+5} = \dfrac{112}{0} = \infty$

(a) $\lim\limits_{\substack{x\to 5\\x<5}} \dfrac{x^3-3\,x+2}{x^2-6\,x+5} = \lim\limits_{\substack{x\to 5\\x<5}} \dfrac{x^3-3\,x+2}{(x-1)\,(x-5)} = \dfrac{112}{0_-} = -\infty$

(b) $\lim\limits_{\substack{x\to 5\\x>5}} \dfrac{x^3-3\,x+2}{x^2-6\,x+5} = \lim\limits_{\substack{x\to 5\\x>5}} \dfrac{x^3-3\,x+2}{(x-1)\,(x-5)} = \dfrac{112}{0_+} = +\infty$

3) $\lim\limits_{x\to 2} \dfrac{x^2-3\,x+2}{x^2-4\,x+4} = \dfrac{0}{0}$: indéterminé

$\lim\limits_{x\to 2} \dfrac{x^2-3\,x+2}{x^2-4\,x+4} = \lim\limits_{x\to 2} \dfrac{(x-1)\,(x-2)}{(x-2)^2} = \lim\limits_{x\to 2} \dfrac{x-1}{x-2} = \dfrac{1}{0} = \infty$

(a) $\lim\limits_{\substack{x\to 2\\x<2}} \dfrac{x^2-3\,x+2}{x^2-4\,x+4} = \lim\limits_{\substack{x\to 2\\x<2}} \dfrac{x-1}{x-2} = \dfrac{1}{0_-} = -\infty$

(b) $\lim\limits_{\substack{x\to 2\\x>2}} \dfrac{x^2-3\,x+2}{x^2-4\,x+4} = \lim\limits_{\substack{x\to 2\\x>2}} \dfrac{x-1}{x-2} = \dfrac{1}{0_+} = +\infty$

4) $\lim\limits_{x\to 1} \dfrac{x^2-3\,x+2}{x^2-4\,x+4} = \dfrac{0}{1} = 0$

5) $\lim\limits_{x\to 1} \dfrac{x^3-3\,x+2}{(x-1)^2} = \dfrac{0}{0}$: indéterminé

Pour factoriser $x^3-3\,x+2$, on utilise le schéma de Horner :

1	0	−3	2
	1	1	−2
1	1	−2	0

On obtient ainsi $x^3-3\,x+2 = (x-1)\,\underbrace{(x^2+x-2)}_{(x-1)\,(x+2)} = (x-1)^2\,(x+2)\,.$

$\lim\limits_{x\to 1} \dfrac{x^3-3\,x+2}{(x-1)^2} = \lim\limits_{x\to 1} \dfrac{(x-1)^2\,(x+2)}{(x-1)^2} = \lim\limits_{x\to 1} x+2 = 3$

6) $\lim\limits_{x\to 1} \left|\dfrac{x^2-1}{x^2-2\,x+1}\right| = \left|\dfrac{0}{0}\right|$: indéterminé

$\lim\limits_{x\to 1} \left|\dfrac{x^2-1}{x^2-2\,x+1}\right| = \lim\limits_{x\to 1} \left|\dfrac{(x-1)\,(x+1)}{(x-1)^2}\right| = \lim\limits_{x\to 1} \left|\dfrac{x+1}{x-1}\right| = \left|\dfrac{2}{0}\right| = +\infty$

7) $\lim\limits_{x \to 0} \dfrac{x^4 - 5\,x^3 + 6\,x^2 + 4\,x - 8}{x^3 - 4\,x^2 + 4\,x} = \dfrac{-8}{0} = \infty$

(a) $\lim\limits_{\substack{x \to 0 \\ x < 0}} \dfrac{x^4 - 5\,x^3 + 6\,x^2 + 4\,x - 8}{x^3 - 4\,x^2 + 4\,x} = \lim\limits_{\substack{x \to 0 \\ x < 0}} \dfrac{x^4 - 5\,x^3 + 6\,x^2 + 4\,x - 8}{x\,(x-2)^2} = \dfrac{-8}{0_-} = +\infty$

(b) $\lim\limits_{\substack{x \to 0 \\ x > 0}} \dfrac{x^4 - 5\,x^3 + 6\,x^2 + 4\,x - 8}{x^3 - 4\,x^2 + 4\,x} = \lim\limits_{\substack{x \to 0 \\ x > 0}} \dfrac{x^4 - 5\,x^3 + 6\,x^2 + 4\,x - 8}{x\,(x-2)^2} = \dfrac{-8}{0_+} = -\infty$

8) $\lim\limits_{x \to 0} \dfrac{x^3 + x^2 - 5\,x}{x^4 - 5\,x^3} = \dfrac{0}{0}$: indéterminé

$\lim\limits_{x \to 0} \dfrac{x^3 + x^2 - 5\,x}{x^4 - 5\,x^3} = \lim\limits_{x \to 0} \dfrac{x\,(x^2 + x - 5}{x^3\,(x-5)} = \lim\limits_{x \to 0} \dfrac{x^2 + x - 5}{x^2\,(x-5)} == \dfrac{-5}{0} = \infty$

(a) $\lim\limits_{\substack{x \to 0 \\ x < 0}} \dfrac{x^3 + x^2 - 5\,x}{x^4 - 5\,x^3} = \lim\limits_{\substack{x \to 0 \\ x < 0}} \dfrac{x^2 + x - 5}{x^2\,(x-5)} = \dfrac{-5}{0_-} = +\infty$

(b) $\lim\limits_{\substack{x \to 0 \\ x > 0}} \dfrac{x^3 + x^2 - 5\,x}{x^4 - 5\,x^3} = \lim\limits_{\substack{x \to 0 \\ x > 0}} \dfrac{x^2 + x - 5}{x^2\,(x-5)} = \dfrac{-5}{0_-} = +\infty$

9) $\lim\limits_{x \to 11} \dfrac{3 - \sqrt{x-2}}{x - 11} = \dfrac{0}{0}$: indéterminé

$\lim\limits_{x \to 11} \dfrac{3 - \sqrt{x-2}}{x - 11} = \lim\limits_{x \to 11} \dfrac{(3 - \sqrt{x-2})\,(3 + \sqrt{x-2})}{(x-11)\,(3 + \sqrt{x-2})} =$

$\lim\limits_{x \to 11} \dfrac{\overbrace{9 - (x-2)}^{-(x-11)}}{(x-11)\,(3 + \sqrt{x-2})} = \lim\limits_{x \to 11} \dfrac{-1}{3 + \sqrt{x-2}} = \dfrac{-1}{6}$

10) $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{x^2+1} - 1} = \dfrac{0}{0}$: indéterminé

$\lim\limits_{x \to 0} \dfrac{x}{\sqrt{x^2+1} - 1} = \lim\limits_{x \to 0} \dfrac{x\,(\sqrt{x^2+1} + 1)}{(\sqrt{x^2+1} - 1)\,(\sqrt{x^2+1} + 1)} = \lim\limits_{x \to 0} \dfrac{x\,(\sqrt{x^2+1} + 1)}{\underbrace{(x^2+1) - 1}_{x^2}} =$

$\lim\limits_{x \to 0} \dfrac{\sqrt{x^2+1} + 1}{x} = \dfrac{1}{0} = \infty$

(a) $\lim\limits_{\substack{x \to 0 \\ x < 0}} \dfrac{x}{\sqrt{x^2+1} - 1} = \lim\limits_{\substack{x \to 0 \\ x < 0}} \dfrac{\sqrt{x^2+1} + 1}{x} = \dfrac{1}{0_-} = -\infty$

(b) $\lim\limits_{\substack{x \to 0 \\ x > 0}} \dfrac{x}{\sqrt{x^2+1} - 1} = \lim\limits_{\substack{x \to 0 \\ x > 0}} \dfrac{\sqrt{x^2+1} + 1}{x} = \dfrac{1}{0_+} = +\infty$

11) $\lim\limits_{x\to 2}\dfrac{4-x^2}{3-\sqrt{x^2+5}}=\dfrac{0}{0}$: indéterminé

$\lim\limits_{x\to 2}\dfrac{4-x^2}{3-\sqrt{x^2+5}}=\lim\limits_{x\to 2}\dfrac{(4-x^2)\,(3+\sqrt{x^2+5})}{(3-\sqrt{x^2+5})\,(3+\sqrt{x^2+5})}=$

$\lim\limits_{x\to 2}\dfrac{(4-x^2)\,(3+\sqrt{x^2+5})}{\underbrace{9-(x^2+5)}_{4-x^2}}=\lim\limits_{x\to 2}3+\sqrt{x^2+5}=6$

12) $\lim\limits_{\substack{x\to 2\\ x>2}}\dfrac{x^3-3\,x-2}{\sqrt{x-2}}=\dfrac{0}{0}$: indéterminé

Factorisons $x^3-3\,x-2$ à l'aide du schéma de Horner :

1	0	−3	−2
	2	4	2
1	2	1 ‖	0

Dès lors $x^3-3\,x-2=(x-2)\,(x^2+2\,x+1)=(x-2)\,(x+1)^2$.

$\lim\limits_{\substack{x\to 2\\ x>2}}\dfrac{x^3-3\,x-2}{\sqrt{x-2}}=\lim\limits_{\substack{x\to 2\\ x>2}}\dfrac{(x^3-3\,x-2)\,\sqrt{x-2}}{x-2}=\lim\limits_{\substack{x\to 2\\ x>2}}\dfrac{(x-2)\,(x+1)^2\,\sqrt{x-2}}{x-2}=$
$\lim\limits_{\substack{x\to 2\\ x>2}}(x+1)\,\sqrt{x-2}=0$

Remarque : $\lim\limits_{\substack{x\to 2\\ x<2}}\dfrac{x^3-3\,x-2}{\sqrt{x-2}}$ n'existe pas, car la fonction $f(x)=\dfrac{x^3-3\,x-2}{\sqrt{x-2}}$ n'est définie que sur $]2\,;+\infty[$.