3.8

1) (a) $\lim\limits_{x\to-\infty}\dfrac{1}{x}=\lim\limits_{\substack{x\to0\\x<0}}\dfrac{1}{\dfrac{1}{x}}=\lim\limits_{\substack{x\to0\\x<0}}x=0$

(b) $\lim\limits_{x\to+\infty}\dfrac{1}{x}=\lim\limits_{\substack{x\to0\\x>0}}\dfrac{1}{\dfrac{1}{x}}=\lim\limits_{\substack{x\to0\\x>0}}x=0$

2) (a) $\lim\limits_{x\to-\infty}-2\,x^2+x+1=\lim\limits_{\substack{x\to0\\x<0}}-2\,(\frac{1}{x})^2+\frac{1}{x}+1=\lim\limits_{\substack{x\to0\\x<0}}\dfrac{-2+x+x^2}{x^2}=$
$\dfrac{-2}{0_+}=-\infty$

(b) $\lim\limits_{x\to+\infty}-2\,x^2+x+1=\lim\limits_{\substack{x\to0\\x>0}}-2\,(\frac{1}{x})^2+\frac{1}{x}+1=\lim\limits_{\substack{x\to0\\x>0}}\dfrac{-2+x+x^2}{x^2}=$
$\dfrac{-2}{0_+}=-\infty$

3) (a) $\lim\limits_{x\to-\infty}-x^3+x^2-x+1=\lim\limits_{\substack{x\to0\\x<0}}-(\frac{1}{x})^3+(\frac{1}{x})^2-\frac{1}{x}+1=\lim\limits_{\substack{x\to0\\x<0}}\dfrac{-1+x-x^2+x^3}{x^3}=$
$\dfrac{-1}{0_-}=+\infty$

(b) $\lim\limits_{x\to+\infty}-x^3+x^2-x+1=\lim\limits_{\substack{x\to0\\x>0}}-(\frac{1}{x})^3+(\frac{1}{x})^2-\frac{1}{x}+1=\lim\limits_{\substack{x\to0\\x>0}}\dfrac{-1+x-x^2+x^3}{x^3}=$
$\dfrac{-1}{0_+}=-\infty$

4) (a) $\lim\limits_{x\to-\infty}\dfrac{x^2-1}{3\,x^2-4}=\lim\limits_{\substack{x\to0\\x<0}}\dfrac{(\frac{1}{x})^2-1}{3\,(\frac{1}{x})^2-4}=\lim\limits_{\substack{x\to0\\x<0}}\dfrac{\dfrac{1-x^2}{x^2}}{\dfrac{3-4\,x^2}{x^2}}=\lim\limits_{\substack{x\to0\\x<0}}\dfrac{1-x^2}{3-4\,x^2}$
$=\dfrac{1}{3}$

(b) $\lim\limits_{x\to+\infty}\dfrac{x^2-1}{3\,x^2-4}=\lim\limits_{\substack{x\to0\\x>0}}\dfrac{(\frac{1}{x})^2-1}{3\,(\frac{1}{x})^2-4}=\lim\limits_{\substack{x\to0\\x>0}}\dfrac{\dfrac{1-x^2}{x^2}}{\dfrac{3-4\,x^2}{x^2}}=\lim\limits_{\substack{x\to0\\x>0}}\dfrac{1-x^2}{3-4\,x^2}$
$=\dfrac{1}{3}$

5) (a) $\lim\limits_{x\to-\infty}\dfrac{x^2-1}{3\,x+1}=\lim\limits_{\substack{x\to0\\x<0}}\dfrac{(\frac{1}{x})^2-1}{3\cdot\frac{1}{x}+1}=\lim\limits_{\substack{x\to0\\x<0}}\dfrac{\dfrac{1-x^2}{x^2}}{\dfrac{1+x}{x}}=$
$\lim\limits_{\substack{x\to0\\x<0}}\dfrac{(1-x)\,(1+x)}{x^2}\cdot\dfrac{x}{1+x}=\lim\limits_{\substack{x\to0\\x<0}}\dfrac{1-x}{x}=\dfrac{1}{0_-}=-\infty$

(b) $\lim\limits_{x\to+\infty} \dfrac{x^2-1}{3\,x+1} = \lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{(\frac{1}{x})^2-1}{3\cdot\frac{1}{x}+1} = \lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{\dfrac{1-x^2}{x^2}}{\dfrac{1+x}{x}} =$

$\lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{(1-x)\,(1+x)}{x^2}\cdot\dfrac{x}{1+x} = \lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{1-x}{x} = \dfrac{1}{0_+} = +\infty$

6) (a) $\lim\limits_{x\to-\infty} x-\sqrt{x^2+2\,x+7} = \lim\limits_{\substack{x\to 0\\ x<0}} \frac{1}{x}-\sqrt{(\frac{1}{x})^2+2\cdot\frac{1}{x}+7} =$

$\lim\limits_{\substack{x\to 0\\ x<0}} \dfrac{1}{x}-\sqrt{\dfrac{1+2\,x+7\,x^2}{x^2}} = \lim\limits_{\substack{x\to 0\\ x<0}} \dfrac{1}{x}-\dfrac{\sqrt{1+2\,x+7\,x^2}}{\underbrace{\sqrt{x^2}}_{|x|}} =$

$\lim\limits_{\substack{x\to 0\\ x<0}} \dfrac{1}{x}-\dfrac{\sqrt{1+2\,x+7\,x^2}}{-x} = \lim\limits_{\substack{x\to 0\\ x<0}} \dfrac{1+\sqrt{1+2\,x+7\,x^2}}{x} = \dfrac{2}{0_-} = -\infty$

(b) $\lim\limits_{x\to+\infty} x-\sqrt{x^2+2\,x+7} = \lim\limits_{\substack{x\to 0\\ x>0}} \frac{1}{x}-\sqrt{(\frac{1}{x})^2+2\cdot\frac{1}{x}+7} =$

$\lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{1}{x}-\sqrt{\dfrac{1+2\,x+7\,x^2}{x^2}} = \lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{1}{x}-\dfrac{\sqrt{1+2\,x+7\,x^2}}{\underbrace{\sqrt{x^2}}_{|x|}} =$

$\lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{1}{x}-\dfrac{\sqrt{1+2\,x+7\,x^2}}{x} = \lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{1-\sqrt{1+2\,x+7\,x^2}}{x}$

$= \dfrac{0}{0}$: indéterminé

$\lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{1-\sqrt{1+2\,x+7\,x^2}}{x} = \lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{(1-\sqrt{1+2\,x+7\,x^2})\,(1+\sqrt{1+2\,x+7\,x^2})}{x\,(1+\sqrt{1+2\,x+7\,x^2})} =$

$\lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{\overbrace{1-(1+2\,x+7\,x^2)}^{x\,(-2-7\,x)}}{x\,(1+\sqrt{1+2\,x+7\,x^2})} = \lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{-2-7\,x}{1+\sqrt{1+2\,x+7\,x^2}} = \dfrac{-2}{2} = -1$