3.9 1) $\lim\limits_{x\to\infty} f(x) =$

$\lim\limits_{x\to\infty} \lambda_n\, x^n + \lambda_{n-1}\, x^{n-1} + \ldots + \lambda_1\, x + \lambda_0 =$

$\lim\limits_{x\to\infty} x^n \left(\lambda_n + \lambda_{n-1}\, \dfrac{x^{n-1}}{x^n} + \ldots + \lambda_1\, \dfrac{x}{x^n} + \lambda_0\, \dfrac{1}{x^n}\right) =$

$\lim\limits_{x\to\infty} x^n \left(\lambda_n + \lambda_{n-1}\, \dfrac{1}{x} + \ldots + \lambda_1\, \dfrac{1}{x^{n-1}} + \lambda_0\, \dfrac{1}{x^n}\right) =$

$\lim\limits_{x\to\infty} x^n \left(\lambda_n + \lambda_{n-1}\, \dfrac{1}{x} + \ldots + \lambda_1 \left(\dfrac{1}{x}\right)^{n-1} + \lambda_0 \left(\dfrac{1}{n}\right)^{n}\right) =$

$\lim\limits_{x\to\infty} x^n \cdot \lim\limits_{x\to\infty} \left(\lambda_n + \lambda_{n-1}\, \dfrac{1}{x} + \ldots + \lambda_1 \left(\dfrac{1}{x}\right)^{n-1} + \lambda_0 \left(\dfrac{1}{n}\right)^{n}\right) =$

$\lim\limits_{x\to\infty} x^n \cdot \left(\lim\limits_{x\to\infty} \lambda_n + \lim\limits_{x\to\infty} \lambda_{n-1}\, \dfrac{1}{x} + \ldots + \lim\limits_{x\to\infty} \lambda_1 \left(\dfrac{1}{x}\right)^{n-1} + \lim\limits_{x\to\infty} \lambda_0 \left(\dfrac{1}{x}\right)^{n}\right) =$

$\lim\limits_{x\to\infty} x^n \cdot \left(\lambda_n \lim\limits_{x\to\infty} 1 + \lambda_{n-1} \lim\limits_{x\to\infty} \dfrac{1}{x} + \ldots + \lambda_1 \lim\limits_{x\to\infty} \left(\dfrac{1}{x}\right)^{n-1} + \lambda_0 \lim\limits_{x\to\infty} \left(\dfrac{1}{x}\right)^{n}\right) =$

$\lim\limits_{x\to\infty} x^n \cdot \left(\lambda_n \underbrace{\lim\limits_{x\to\infty} 1}_{1} + \lambda_{n-1} \underbrace{\lim\limits_{x\to\infty} \dfrac{1}{x}}_{0} + \ldots + \lambda_1 \left(\underbrace{\lim\limits_{x\to\infty} \dfrac{1}{x}}_{0}\right)^{n-1} + \lambda_0 \left(\underbrace{\lim\limits_{x\to\infty} \dfrac{1}{x}}_{0}\right)^{n}\right) =$

$\lim\limits_{x\to\infty} x^n \cdot \lambda_n = \lim\limits_{x\to\infty} \lambda_n\, x^n$

2) $\lim\limits_{x\to\infty} \dfrac{\lambda_n\, x^n + \lambda_{n-1}\, x^{n-1} + \ldots + \lambda_1\, x + \lambda_0}{\mu_m\, x^m + \mu_{m-1}\, x^{m-1} + \ldots + \mu_1\, x + \mu_0} =$

$\dfrac{\lim\limits_{x\to\infty} \lambda_n\, x^n + \lambda_{n-1}\, x^{n-1} + \ldots + \lambda_1\, x + \lambda_0}{\lim\limits_{x\to\infty} \mu_m\, x^m + \mu_{m-1}\, x^{m-1} + \ldots + \mu_1\, x + \mu_0} = \dfrac{\lim\limits_{x\to\infty} \lambda_n\, x^n}{\lim\limits_{x\to\infty} \mu_m\, x^m} = \lim\limits_{x\to\infty} \dfrac{\lambda_n\, x^n}{\mu_m\, x^m}$

(a) Supposons $n < m \iff 0 < m - n$.

$\lim\limits_{x\to\infty} \dfrac{\lambda_n\, x^n}{\mu_m\, x^m} = \lim\limits_{x\to\infty} \dfrac{\lambda_n\, x^n}{\mu_m\, x^n\, x^{m-n}} = \lim\limits_{x\to\infty} \dfrac{\lambda_n}{\mu_m\, x^{m-n}} = \dfrac{\lambda_n}{\mu_m} \lim\limits_{x\to\infty} \dfrac{1}{x^{m-n}}$

$\dfrac{\lambda_n}{\mu_m} \lim\limits_{x\to\infty} \left(\dfrac{1}{x}\right)^{m-n} = \dfrac{\lambda_n}{\mu_m} \cdot 0 = 0$

(b) Supposons $n = m$.

$\lim\limits_{x\to\infty} \dfrac{\lambda_n\, x^n}{\mu_m\, x^m} = \lim\limits_{x\to\infty} \dfrac{\lambda_n}{\mu_m} = \dfrac{\lambda_n}{\mu_m}$

(c) Supposons $n > m \iff n - m > 0$.

$\lim\limits_{x\to\infty} \dfrac{\lambda_n\, x^n}{\mu_m\, x^m} = \lim\limits_{x\to\infty} \dfrac{\lambda_n\, x^m\, x^{n-m}}{\mu_m\, x^m} = \lim\limits_{x\to\infty} \dfrac{\lambda_n\, x^{n-m}}{\mu_m} = \dfrac{\lambda_n}{\mu_m} \lim\limits_{x\to\infty} x^{n-m} =$

$\dfrac{\lambda_n}{\mu_m} \cdot \infty = \infty$