3.3 Posons $\alpha = \log_a(x)$ et $\beta = \log_a(y)$.
Par définition, on a $x = a^\alpha$ et $y = a^\beta$.

1) $\log_a(x\,y) = \log_a(a^\alpha \cdot a^\beta) = \log_a(a^{\alpha+\beta}) = \alpha + \beta = \log_a(x) + \log_a(y)$

2) $\log_a(\frac{1}{y}) = \log_a(\frac{1}{a^\beta}) = \log_a(a^{-\beta}) = -\beta = -\log_a(y)$

3) $\log_a(\frac{x}{y}) = \log_a(\frac{a^\alpha}{a^\beta}) = \log_a(a^{\alpha-\beta}) = \alpha - \beta = \log_a(x) - \log_a(y)$

4) $\log_a(x^p) = \log_a\big((a^\alpha)^p\big) = \log_a(a^{p\,\alpha}) = p\,\alpha = p\,\log_a(x)$