

4.20

1) (a) Supposons $\alpha < \beta$.

$$\min(\alpha, \beta) + \max(\alpha, \beta) = \alpha + \beta$$

(b) Supposons $\alpha = \beta$.

$$\min(\alpha, \beta) + \max(\alpha, \beta) = \alpha + \beta$$

(c) Supposons $\alpha > \beta$.

$$\min(\alpha, \beta) + \max(\alpha, \beta) = \beta + \alpha = \alpha + \beta$$

On obtient ainsi $\max(\alpha, \beta) = \alpha + \beta - \min(\alpha, \beta)$.

$$\begin{aligned} 2) \text{ ppcm}(a, b) &= \frac{a b}{\text{pgcd}(a, b)} = \frac{(p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}) (p_1^{\beta_1} p_2^{\beta_2} \dots p_k^{\beta_k})}{p_1^{\min(\alpha_1, \beta_1)} p_2^{\min(\alpha_2, \beta_2)} \dots p_k^{\min(\alpha_k, \beta_k)}} \\ &= \frac{p_1^{\alpha_1 + \beta_1} p_2^{\alpha_2 + \beta_2} \dots p_k^{\alpha_k + \beta_k}}{p_1^{\min(\alpha_1, \beta_1)} p_2^{\min(\alpha_2, \beta_2)} \dots p_k^{\min(\alpha_k, \beta_k)}} \\ &= p_1^{\alpha_1 + \beta_1 - \min(\alpha_1, \beta_1)} p_2^{\alpha_2 + \beta_2 - \min(\alpha_2, \beta_2)} \dots p_k^{\alpha_k + \beta_k - \min(\alpha_k, \beta_k)} \\ &= p_1^{\max(\alpha_1, \beta_1)} p_2^{\max(\alpha_2, \beta_2)} \dots p_k^{\max(\alpha_k, \beta_k)} \end{aligned}$$