10.10 1) $\Big(\mathrm{G}\big(f(x)\big)\Big)' = \mathrm{G}'\big(f(x)\big)\cdot f'(x) = g\big(f(x)\big)\cdot f'(x)$

Ainsi $\mathrm{G}\big(f(x)\big)$ est une primitive de $g\big(f(x)\big)\cdot f'(x)$.

On obtient donc la formule $\displaystyle\int g\big(f(x)\big)\cdot f'(x)\,dx = \mathrm{G}\big(f(x)\big)$.

2) (a) si $g(x) = \frac{1}{x}$, alors $\mathrm{G}(x) = \ln\big(|x|\big)$:

$$\int \frac{f'(x)}{f(x)}\,dx = \int \frac{1}{f(x)}\cdot f'(x)\,dx = \int g\big(f(x)\big)\cdot f'(x)\,dx$$
$$= \mathrm{G}\big(f(x)\big) = \ln\Big(\big|f(x)\big|\Big)$$

(b) si $g(x) = e^x$, alors $\mathrm{G}(x) = e^x$:

$$\int e^{f(x)}\cdot f'(x)\,dx = \int g\big(f(x)\big)\cdot f'(x)\,dx = \mathrm{G}\big(f(x)\big) = e^{f(x)}$$

(c) si $g(x) = \sin(x)$, alors $\mathrm{G}(x) = -\cos(x)$:

$$\int \sin\big(f(x)\big)\cdot f'(x)\,dx = \int g\big(f(x)\big)\cdot f'(x)\,dx = \mathrm{G}\big(f(x)\big) = -\cos\big(f(x)\big)$$

(d) si $g(x) = \cos(x)$, alors $\mathrm{G}(x) = \sin(x)$:

$$\int \cos\big(f(x)\big)\cdot f'(x)\,dx = \int g\big(f(x)\big)\cdot f'(x)\,dx = \mathrm{G}\big(f(x)\big) = \sin\big(f(x)\big)$$