10.11 1) $\displaystyle\int \frac{1}{2\,x-5}\,dx = \int \frac{1}{2\,x-5}\cdot 2\cdot \tfrac{1}{2}\,dx = \tfrac{1}{2}\int \frac{1}{2\,x-5}\cdot 2\,dx$

$$= \tfrac{1}{2}\int \frac{1}{2\,x-5}\cdot (2\,x-5)'\,dx = \tfrac{1}{2}\,\ln\big(|2\,x-5|\big)+c$$

2) $\displaystyle\int \frac{1}{3\,x+1}\,dx = \int \frac{1}{3\,x+1}\cdot 3\cdot \tfrac{1}{3}\,dx = \tfrac{1}{3}\int \frac{1}{3\,x+1}\cdot 3\,dx$

$$= \tfrac{1}{3}\int \frac{1}{3\,x+1}\cdot (3\,x+1)'\,dx = \tfrac{1}{3}\,\ln\big(|3\,x+1|\big)+c$$

3) $\displaystyle\int \frac{1}{1-2\,x}\,dx = \int \frac{1}{1-2\,x}\cdot (-2)\cdot(-\tfrac{1}{2})\,dx = -\tfrac{1}{2}\int \frac{1}{1-2\,x}\cdot (-2)\,dx$

$$= -\tfrac{1}{2}\int \frac{1}{1-2\,x}\cdot (1-2\,x)'\,dx = -\tfrac{1}{2}\,\ln\big(|1-2\,x|\big)+c$$

4) $\displaystyle\int \frac{x-1}{x^2-2\,x+4}\,dx = \int \frac{1}{x^2-2\,x+4}\cdot (2\,x-2)\cdot \tfrac{1}{2}\,dx$

$$= \tfrac{1}{2}\int \frac{1}{x^2-2\,x+4}\cdot (2\,x-2)\,dx$$

$$= \tfrac{1}{2}\int \frac{1}{x^2-2\,x+4}\cdot (x^2-2\,x+4)'\,dx$$

$$= \tfrac{1}{2}\,\ln\big(|x^2-2\,x+4|\big) = \tfrac{1}{2}\,\ln(x^2-2\,x+4)+c$$

en effet $x^2-2\,x+4>0$ pour tout $x\in\mathbb{R}$, car $\Delta=(-2)^2-4\cdot1\cdot4=-12<0$

5) $\displaystyle\int \frac{3\,x}{x^2+1}\,dx = \int \frac{1}{x^2+1}\cdot (2\,x)\cdot \tfrac{3}{2}\,dx = \tfrac{3}{2}\int \frac{1}{x^2+1}\cdot (2\,x)\,dx$

$$= \tfrac{3}{2}\int \frac{1}{x^2+1}\cdot (x^2+1)'\,dx = \tfrac{3}{2}\,\ln\big(|x^2+1|\big) = \tfrac{3}{2}\,\ln(x^2+1)+c$$

en effet $x^2+1\geqslant 1>0$ quel que soit $x\in\mathbb{R}$

6) $\displaystyle\int \frac{4\,x+2}{x^2+x+1}\,dx = \int \frac{1}{x^2+x+1}\cdot (2\,x+1)\cdot 2\,dx$

$$= 2\int \frac{1}{x^2+x+1}\cdot (2\,x+1)\,dx$$

$$= 2\int \frac{1}{x^2+x+1}\cdot (x^2+x+1)'\,dx$$

$$= 2\,\ln\big(|x^2+x+1|\big) = 2\,\ln(x^2+x+1)+c$$

en effet $x^2+x+1>0$ pour tout $x\in\mathbb{R}$, vu que $\Delta=1^2-4\cdot1\cdot1=-3<0$

7) $\displaystyle\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = \int \frac{1}{\cos(x)} \cdot \big(-\sin(x)\big) \cdot (-1)\,dx$

$\displaystyle = -\int \frac{1}{\cos(x)} \cdot \big(-\sin(x)\big)\,dx = -\int \frac{1}{\cos(x)} \cdot \big(\cos(x)\big)'\,dx$

$\displaystyle = -\ln\Big(\big|\cos(x)\big|\Big) + c$

8) $\displaystyle\int \cot(x)\,dx = \int \frac{\cos(x)}{\sin(x)}\,dx = \int \frac{1}{\sin(x)} \cdot \cos(x)\,dx = \int \frac{1}{\sin(x)} \cdot \big(\sin(x)\big)'\,dx$

$\displaystyle = \ln\Big(\big|\sin(x)\big|\Big) + c$

9) $\displaystyle\int e^{5x}\,dx = \int e^{5x} \cdot 5 \cdot \tfrac{1}{5}\,dx = \tfrac{1}{5} \int e^{5x} \cdot 5\,dx = \tfrac{1}{5} \int e^{5x} \cdot (5\,x)'\,dx = \tfrac{1}{5}\,e^{5x} + c$

10) $\displaystyle\int 2\,e^{3\,x}\,dx = 2 \int e^{3\,x}\,dx = 2 \int e^{3\,x} \cdot 3 \cdot \tfrac{1}{3}\,dx = 2 \cdot \tfrac{1}{3} \int e^{3x} \cdot 3\,dx$

$\displaystyle = \tfrac{2}{3} \int e^{3x} \cdot (3\,x)'\,dx = \tfrac{2}{3}\,e^{3x} + c$

11) $\displaystyle\int e^{2x+1}\,dx = \int e^{2\,x+1} \cdot 2 \cdot \tfrac{1}{2}\,dx = \tfrac{1}{2} \int e^{2\,x+1} \cdot 2\,dx$

$\displaystyle = \tfrac{1}{2} \int e^{2\,x+1} \cdot (2\,x+1)'\,dx = \tfrac{1}{2}\,e^{2\,x+1} + c$

12) $\displaystyle\int e^{-3x}\,dx = \int e^{-3x} \cdot (-3) \cdot (-\tfrac{1}{3})\,dx = -\tfrac{1}{3} \int e^{-3x} \cdot (-3)\,dx$

$\displaystyle = -\tfrac{1}{3} \int e^{-3x} \cdot (-3\,x)'\,dx = -\tfrac{1}{3}\,e^{-3x} + c$

13) $\displaystyle\int \sin(3\,x)\,dx = \int \sin(3\,x) \cdot 3 \cdot \tfrac{1}{3}\,dx = \tfrac{1}{3} \int \sin(3\,x) \cdot 3\,dx$

$\displaystyle = \tfrac{1}{3} \int \sin(3\,x) \cdot (3\,x)'\,dx = \tfrac{1}{3}\,\big(-\cos(3\,x)\big) = -\tfrac{1}{3}\,\cos(3\,x) + c$

14) $\displaystyle\int \frac{\cos(4\,x)}{2}\,dx = \int \cos(4\,x) \cdot \tfrac{1}{2}\,dx = \int \cos(4\,x) \cdot 4 \cdot \tfrac{1}{4} \cdot \tfrac{1}{2}\,dx$

$\displaystyle = \tfrac{1}{4} \cdot \tfrac{1}{2} \int \cos(4\,x) \cdot 4\,dx = \tfrac{1}{8} \int \cos(4\,x) \cdot (4\,x)'\,dx$

$\displaystyle = \tfrac{1}{8}\,\sin(4\,x) + c$

15) $\displaystyle\int \big(\sin(5\,x) - 6\,\cos(3\,x+1)\big)\,dx = \int \sin(5\,x)\,dx - 6\int \cos(3\,x+1)\,dx =$

$\displaystyle\int \sin(5\,x)\cdot 5\cdot \tfrac{1}{5}\,dx - 6\int \cos(3\,x+1)\cdot 3\cdot \tfrac{1}{3}\,dx =$

$\displaystyle\tfrac{1}{5}\int \sin(5\,x)\cdot 5\,dx - 6\cdot\tfrac{1}{3}\int \cos(3\,x+1)\cdot 3\,dx =$

$\displaystyle\tfrac{1}{5}\int \sin(5\,x)\cdot (5\,x)'\,dx - 2\int \cos(3\,x+1)\cdot (3\,x+1)'\,dx =$

$-\tfrac{1}{5}\,\cos(5\,x) - 2\,\sin(3\,x+1) + c$

16) $\displaystyle\int x\,\cos(x^2)\,dx = \int \cos(x^2)\cdot(2\,x)\cdot\tfrac{1}{2}\,dx = \tfrac{1}{2}\int \cos(x^2)\cdot(2\,x)\,dx$

$\displaystyle= \tfrac{1}{2}\int \cos(x^2)\cdot(x^2)'\,dx = \tfrac{1}{2}\,\sin(x^2) + c$