10.7

1) $\displaystyle\int (x+3)^3\,dx = \int \underbrace{(x+3)^3}_{f^3}\cdot\underbrace{1}_{f'}\,dx = \tfrac{1}{4}\,(x+3)^4 + c$

2) $\displaystyle\int (2\,x-1)^2\,dx = \int \underbrace{(2\,x-1)^2}_{f^2}\cdot\underbrace{2}_{f'}\cdot\tfrac{1}{2}\,dx = \tfrac{1}{2}\int \underbrace{(2\,x-1)^2}_{f^2}\cdot\underbrace{2}_{f'}\,dx$

$= \tfrac{1}{2}\cdot\tfrac{1}{3}\,(2\,x-1)^3 = \tfrac{1}{6}\,(2\,x-1)^3 + c$

3) $\displaystyle\int (7\,x-2)^5\,dx = \int \underbrace{(7\,x-2)^5}_{f^5}\cdot\underbrace{7}_{f'}\cdot\tfrac{1}{7}\,dx = \tfrac{1}{7}\int \underbrace{(7\,x-2)^5}_{f^5}\cdot\underbrace{7}_{f'}\,dx$

$= \tfrac{1}{7}\cdot\tfrac{1}{6}\,(7\,x-2)^6 = \tfrac{1}{42}\,(7\,x-2)^6 + c$

4) $\displaystyle\int (3\,x+2)^6\,dx = \int \underbrace{(3\,x+2)^6}_{f^6}\cdot\underbrace{3}_{f'}\cdot\tfrac{1}{3}\,dx = \tfrac{1}{3}\int \underbrace{(3\,x+2)^6}_{f^6}\cdot\underbrace{3}_{f'}\,dx$

$= \tfrac{1}{3}\cdot\tfrac{1}{7}\,(3\,x+2)^7 = \tfrac{1}{21}\,(3\,x+2)^7 + c$

5) $\displaystyle\int (3\,x^2+x)^3\,(6\,x+1)\,dx = \int \underbrace{(3\,x^2+x)^3}_{f^3}\,\underbrace{(3\,x^2+x)'}_{f'}\,dx = \frac{1}{4}\,(3\,x^2+x)^4+c$

6) $\displaystyle\int (4\,x^2-5\,x)^2\,(16\,x-10)\,dx = \int (4\,x^2-5\,x)^2\,(8\,x-5)\cdot 2\,dx$

$\displaystyle= 2\int (4\,x^2-5\,x)^2\,(8\,x-5)\,dx$

$\displaystyle= 2\int (4\,x^2-5\,x)^2\,(4\,x^2-5\,x)'\,dx$

$= 2\cdot\tfrac{1}{3}\,(4\,x^2-5\,x)^3 = \tfrac{2}{3}\,(4\,x^2-5\,x)^3 + c$

7) $\displaystyle\int x\,(4\,x^2+3)^4\,dx = \int (4\,x^2+3)^4\,8\,x\cdot\tfrac{1}{8}\,dx = \tfrac{1}{8}\int (4\,x^2+3)^4\,8\,x\,dx$

$\displaystyle= \tfrac{1}{8}\int (4\,x^2+3)^4\,(4\,x^2+3)'\,dx = \tfrac{1}{8}\cdot\tfrac{1}{5}\,(4\,x^2+3)^5$

$= \tfrac{1}{40}\,(4\,x^2+3)^5 + c$

8) $\displaystyle\int (x^2+2\,x)\,(x^3+3\,x^2-5)^2\,dx = \int (x^3+3\,x^2-5)^2\,(x^2+2\,x)\cdot 3\cdot\tfrac{1}{3}\,dx$

$\displaystyle= \tfrac{1}{3}\int (x^3+3\,x^2-5)^2\,(3\,x^2+6\,x)\,dx$

$\displaystyle= \tfrac{1}{3}\int (x^3+3\,x^2-5)^2\,(x^3+3\,x^2-5)'\,dx$

$= \tfrac{1}{3}\cdot\tfrac{1}{3}\,(x^3+3\,x^2-5)^3 = \tfrac{1}{9}\,(x^3+3\,x^2-5)^3+c$

9) $$\int \frac{2\,x+1}{(x^2+x+3)^2}\,dx = \int \frac{1}{(x^2+x+3)^2}\cdot(2\,x+1)\,dx$$
$$= \int (x^2+x+3)^{-2}\cdot(x^2+x+3)'\,dx$$
$$= \tfrac{1}{-1}\,(x^2+x+3)^{-1} = -\frac{1}{x^2+x+3}+c$$

10) $$\int \frac{3\,x^2}{(1+2\,x^3)^2}\,dx = \int \frac{1}{(1+2\,x^3)^2}\cdot 3\,x^2\cdot 2\cdot\tfrac{1}{2}\,dx$$
$$= \tfrac{1}{2}\int \frac{1}{(1+2\,x^3)^2}\cdot 6\,x^2\,dx$$
$$= \tfrac{1}{2}\int (1+2\,x^3)^{-2}\cdot(1+2\,x^3)'\,dx$$
$$= \tfrac{1}{2}\cdot\tfrac{1}{-1}\,(1+2\,x^3)^{-1} = -\frac{1}{2\,(1+2\,x^3)}+c$$

11) $$\int (3\,x^2+1)\,\sqrt{x^3+x+2}\,dx = \int (x^3+x+2)^{\frac{1}{2}}\,(3\,x^2+1)\,dx$$
$$= \int (x^3+x+2)^{\frac{1}{2}}\,(x^3+x+2)'\,dx$$
$$= \frac{1}{\frac{3}{2}}\,(x^3+x+2)^{\frac{3}{2}} = \tfrac{2}{3}\,\sqrt{(x^3+x+2)^3}$$
$$= \tfrac{2}{3}\,(x^3+x+2)\,\sqrt{x^3+x+2}+c$$

12) $$\int (2\,x-5)\,\sqrt{x^2-5\,x+6}\,dx = \int (x^2-5\,x+6)^{\frac{1}{2}}\,(2\,x-5)\,dx$$
$$= \int (x^2-5\,x+6)^{\frac{1}{2}}\,(x^2-5\,x+6)'\,dx$$
$$= \frac{1}{\frac{3}{2}}\,(x^2-5\,x+6)^{\frac{3}{2}} = \tfrac{2}{3}\,\sqrt{(x^2-5\,x+6)^3}$$
$$= \tfrac{2}{3}\,(x^2-5\,x+6)\,\sqrt{x^2-5\,x+6}+c$$

13) $$\int \frac{1}{\sqrt{3\,x+1}}\,dx = \int (3\,x+1)^{-\frac{1}{2}}\,dx = \int (3\,x+1)^{-\frac{1}{2}}\cdot 3\cdot\tfrac{1}{3}\,dx$$
$$= \tfrac{1}{3}\int (3\,x+1)^{\frac{1}{2}}\,(3\,x+1)'\,dx = \tfrac{1}{3}\cdot\frac{1}{\frac{1}{2}}\,(3\,x+1)^{\frac{1}{2}}$$
$$= \tfrac{1}{3}\cdot 2\,\sqrt{3\,x+1} = \tfrac{2}{3}\,\sqrt{3\,x+1}+c$$

14) $$\int \frac{x+1}{\sqrt{x^2+2\,x}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x}}\,(x+1)\,dx$$
$$= \int (x^2+2\,x)^{-\frac{1}{2}}\,(x+1)\cdot 2\cdot\tfrac{1}{2}\,dx$$
$$= \tfrac{1}{2}\int (x^2+2\,x)^{-\frac{1}{2}}\,(2\,x+2)\,dx$$

$$= \tfrac{1}{2} \int (x^2 + 2\,x)^{-\frac{1}{2}}\,(x^2 + 2\,x)'\,dx = \tfrac{1}{2} \cdot \frac{1}{\frac{1}{2}}\,(x^2 + 2\,x)^{\frac{1}{2}}$$
$$= \sqrt{x^2 + 2\,x} + c$$

15) $$\int \frac{3\,x^2}{\sqrt{9 + x^3}}\,dx = \int \frac{1}{9 + x^3} \cdot 3\,x^2\,dx = \int (9 + x^3)^{-\frac{1}{2}} \cdot 3\,x^2\,dx$$
$$= \int (9+x^3)^{-\frac{1}{2}}\,(9+x^3)'\,dx = \frac{1}{\frac{1}{2}}\,(9+x^3)^{\frac{1}{2}} = 2\,\sqrt{9 + x^3} + c$$

16) $$\int \frac{3\,x^2}{\sqrt{5\,x^3 + 8}}\,dx = \int \frac{1}{\sqrt{5\,x^3 + 8}} \cdot 3\,x^2 \cdot 5 \cdot \tfrac{1}{5}\,dx = \tfrac{1}{5} \int (5\,x^3 + 8)^{-\frac{1}{2}} \cdot 15\,x^2\,dx$$
$$= \tfrac{1}{5} \int (5\,x^3 + 8)^{-\frac{1}{2}}\,(5\,x^3 + 8)'\,dx = \tfrac{1}{5} \cdot \frac{1}{\frac{1}{2}}\,(5\,x^3 + 8)^{\frac{1}{2}}$$
$$= \tfrac{2}{5}\,\sqrt{5\,x^3 + 8} + c$$

17) $$\int \cos(x)\,\sqrt{\sin(x)}\,dx = \int \big(\sin(x)\big)^{\frac{1}{2}}\,\cos(x)\,dx = \int \big(\sin(x)\big)^{\frac{1}{2}}\,\big(\sin(x)\big)'\,dx$$
$$= \frac{1}{\frac{3}{2}}\,\big(\sin(x)\big)^{\frac{3}{2}} = \tfrac{2}{3}\,\sqrt{\big(\sin(x)\big)^3} = \tfrac{2}{3}\,\sin(x)\,\sqrt{\sin(x)} + c$$

18) $$\int \sin(x)\,\cos^4(x)\,dx = \int \cos^4(x)\,\big(-\sin(x)\big) \cdot (-1)\,dx$$
$$= (-1) \int \cos^4(x)\,\big(\cos(x)\big)'\,dx = -\tfrac{1}{5}\,\cos^5(x) + c$$

19) $$\big(\cos(\tfrac{x}{2})\big)' = -\sin(\tfrac{x}{2}) \cdot (\tfrac{x}{2})' = -\tfrac{1}{2}\,\sin(\tfrac{x}{2})$$
$$\int \cos^2(\tfrac{x}{2})\,\sin(\tfrac{x}{2})\,dx = \int \cos^2(\tfrac{x}{2})\,\big(-\tfrac{1}{2}\,\sin(\tfrac{x}{2})\big) \cdot (-2)\,dx$$
$$= -2 \int \cos^2(\tfrac{x}{2})\,\big(\cos(\tfrac{x}{2})\big)'\,dx = -2 \cdot \tfrac{1}{3}\,\cos^3(\tfrac{x}{2})$$
$$= -\tfrac{2}{3}\,\cos^3(\tfrac{x}{2}) + c$$

20) $$\int \cos(x) - \sin^2(x)\,\cos(x)\,dx = \int \cos(x)\,dx - \int \sin^2(x)\,\cos(x)\,dx$$
$$= \int \cos(x)\,dx - \int \sin^2(x)\,\big(\sin(x)\big)'\,dx$$
$$= \sin(x) - \tfrac{1}{3}\,\sin^3(x) + c$$

$$21)\ \int \frac{\sin(x)}{\big(1+\cos(x)\big)^2}\,dx = \int \frac{1}{\big(1+\cos(x)\big)^2}\ \sin(x)\,dx$$
$$= \int \big(1+\cos(x)\big)^{-2}\,\big(-\sin(x)\big)\cdot(-1)\,dx$$
$$= (-1)\ \int \big(1+\cos(x)\big)^{-2}\,\big(1+\cos(x)\big)'\,dx$$
$$= (-1)\cdot\tfrac{1}{-1}\,\big(1+\cos(x)\big)^{-1} = \frac{1}{1+\cos(x)} + c$$

$$22)\ \int \frac{\cos(x)}{\big(4\,\sin(x)-1\big)^3}\,dx = \int \frac{1}{\big(4\,\sin(x)-1\big)^3}\ \cos(x)\,dx$$
$$= \int \big(4\,\sin(x)-1\big)^{-3}\cdot 4\,\cos(x)\cdot\tfrac{1}{4}\,dx$$
$$= \tfrac{1}{4}\ \int \big(4\,\sin(x)-1\big)^{-3}\,\big(4\,\sin(x)-1\big)'\,dx$$
$$= \tfrac{1}{4}\cdot\tfrac{1}{-2}\,\big(4\,\sin(x)-1\big)^{-2} = -\frac{1}{8\,\big(4\,\sin(x)-1\big)^2} + c$$