

1.8

$$1) \frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{2}-1} = \frac{1(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} + \frac{1(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$$

$$= \frac{\sqrt{2}-1}{1} + \frac{\sqrt{2}+1}{1} = 2\sqrt{2}$$

$$2) \frac{\sqrt{3}-1}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}+1}{\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

$$= \frac{6-2\sqrt{6}}{1} = 6-2\sqrt{6}$$

$$3) \frac{10+3\sqrt{21}}{\sqrt{3}+\sqrt{7}} + \frac{10-2\sqrt{21}}{\sqrt{3}-\sqrt{7}}$$

$$= \frac{(10+3\sqrt{21})(\sqrt{3}-\sqrt{7})}{(\sqrt{3}+\sqrt{7})(\sqrt{3}-\sqrt{7})} + \frac{(10-2\sqrt{21})(\sqrt{3}+\sqrt{7})}{(\sqrt{3}-\sqrt{7})(\sqrt{3}+\sqrt{7})}$$

$$= \frac{10\sqrt{3}-10\sqrt{7}+9\sqrt{7}-21\sqrt{3}}{-4} + \frac{10\sqrt{3}+10\sqrt{7}-6\sqrt{7}-14\sqrt{3}}{-4}$$

$$= \frac{-15\sqrt{3}+3\sqrt{7}}{-4} = \frac{15\sqrt{3}-3\sqrt{7}}{4}$$

$$4) \frac{2}{3-\sqrt{5}} + \frac{3\sqrt{2}}{2-\sqrt{2}} = \frac{2(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} + \frac{3\sqrt{2}(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}$$

$$= \frac{6+2\sqrt{5}}{4} + \frac{6\sqrt{2}+6}{2}$$

$$= \frac{6+2\sqrt{5}}{4} + \frac{12\sqrt{2}+12}{4} = \frac{18+12\sqrt{2}+2\sqrt{5}}{4}$$

$$= \frac{2(9+6\sqrt{2}+\sqrt{5})}{4} = \frac{9+6\sqrt{2}+\sqrt{5}}{2}$$

$$5) \frac{4}{\sqrt{7}-\sqrt{5}} + \frac{1}{\sqrt{6}-\sqrt{7}} + \frac{6}{\sqrt{7}-5}$$

$$= \frac{4(\sqrt{7}+\sqrt{5})}{(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})} + \frac{1(\sqrt{6}+\sqrt{7})}{(\sqrt{6}-\sqrt{7})(\sqrt{6}+\sqrt{7})} + \frac{6(\sqrt{7}+5)}{(\sqrt{7}-5)(\sqrt{7}+5)}$$

$$= \frac{4(\sqrt{7}+\sqrt{5})}{2} + \frac{\sqrt{6}+\sqrt{7}}{-1} + \frac{6(\sqrt{7}+5)}{-18}$$

$$= 2(\sqrt{7}+\sqrt{5}) - (\sqrt{6}+\sqrt{7}) - \frac{\sqrt{7}+5}{3}$$

$$= 2\sqrt{7} + 2\sqrt{5} - \sqrt{6} - \sqrt{7} - \frac{1}{3}\sqrt{7} - \frac{5}{3}$$

$$= -\frac{5}{3} + 2\sqrt{5} - \sqrt{6} + \frac{2}{3}\sqrt{7}$$

$$\begin{aligned}
6) \quad & \frac{1}{\sqrt{3}-\sqrt{2}} - \frac{7}{3-\sqrt{2}} + \frac{5}{2-\sqrt{3}} \\
&= \frac{1(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} - \frac{7(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} + \frac{5(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} \\
&= \frac{\sqrt{3}+\sqrt{2}}{1} - \frac{7(3+\sqrt{2})}{7} + \frac{5(2+\sqrt{3})}{1} \\
&= \sqrt{3} + \sqrt{2} - 3 - \sqrt{2} + 10 + 5\sqrt{3} \\
&= 7 + 6\sqrt{3}
\end{aligned}$$