

$$\begin{aligned} 4.5 \quad v_{n+1} - v_n &= (u_{n+2}^2 - u_{n+1}^2) - (u_{n+1}^2 - u_n^2) = u_{n+2}^2 - 2u_{n+1}^2 + u_n^2 = \\ &= (u_n + 2r)^2 - 2(u_n + r)^2 + u_n^2 \\ &= u_n^2 + 4u_n r + 4r^2 - 2u_n^2 - 4u_n r - 2r^2 + u_n^2 \\ &= 2r^2 \end{aligned}$$

La suite $(v_n)_{n \in \mathbb{N}}$ est donc une suite arithmétique de raison $2r^2$.